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Adaptive Decoupled Fuzzy Sliding-Mode Control of a Nonlinear Aeroelastic System

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I. Introduction

AEROELASTIC systems may exhibit a variety of responses including limit-cycle oscillation, flutter, internal resonance, and even chaotic motion. For certain wing store configurations, researchers have found that fighter aircraft undergo undesirable limit-cycle oscillations. Active control of aeroelastic systems is, thus, of considerable interest. Whereas a number of nonlinear and adaptive control algorithms for aeroelastic nonlinear systems have been proposed,[1–3] most of them consider only the single-input/single-output aeroelastic control problem, that is, the flap deflection has been used to control the pitch angle and the plunge displacement has been shown to be asymptotically stable without control; or the plunge displacement has been controlled by the flap deflection and the pitch angle has been shown to be asymptotically stable without control.[1–3] Unfortunately, most of these design methods require a system model and complex design procedures. Modeling of an aeroelastic system is a work of approximation because the precise model of an aeroelastic system is difficult to formulate.

Sliding-mode control (SMC) is a robust design methodology that is developed in the sense of Lyapunov's function.[4] The main advantage of SMC is that the system uncertainties can be handled under the invariance characteristics of the system's sliding condition. The main disadvantage is chattering of the control signal. Recently, there has been much research on the design of fuzzy logic control (FLC) system based on SMC.[5,6] Fuzzy SLC (FSMC) combines the advantages of both FLC and SMC and can reduce the chattering of the control system (compared to SMC). By the introduction of an intermediate variable, a decoupled FSMC design method has been proposed to achieve decoupling performance of a class of nonlinear coupled systems.[7] In Ref. 8, a hybrid FSMC has been proposed to control an aeroelastic system. However, the decoupled FSMC methods presented in Refs. 7 and 8 have not discussed the stability problem.

In this Note, an adaptive decoupled FSMC (ADFSMC) for an aeroelastic system is derived. The aeroelastic model describes the nonlinear plunge and pitch motions of a wing section, using a single trailing-edge flap as the control input. The approach involves first dividing the aeroelastic system into two subsystems, and then constructing two sliding surfaces using the state variables of the decoupled system. An intermediate variable is introduced to incorporate these two sliding surfaces. Finally, an ADFSMC system is designed to control the plunge and pitch motions (simultaneously).

II. Decoupling Control of Nonlinear Aeroelastic System

The equations of motion for the nonlinear aeroelastic system can be obtained.[1,2]

Subsystem A:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f_1(\boldsymbol{x}) + g_1 u \tag{1a}$$

Subsystem B:

$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = f_2(\boldsymbol{x}) + g_2 u \tag{1b}$$

where h and α are the plunge displacement and pitch angle, respectively,

$$\boldsymbol{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [h \quad \dot{h} \quad \alpha \quad \dot{\alpha}]^T$$
$$f_1(\boldsymbol{x}) = -k_1 x_1 - \left[k_2 U^2 + p(x_2)\right]x_2 - c_1 x_3 - c_2 x_4$$
$$f_2(\boldsymbol{x}) = -k_3 x_1 - \left[k_4 U^2 + q(x_2)\right]x_2 - c_3 x_3 - c_4 x_4$$
$$g_1 = -\left(I_\alpha \rho b c_{l\beta} + m_W x_\alpha \rho b^3 c_{m\beta}\right)U^2 \Big/ d$$

$g_2 = (m_w x_\alpha \rho b^2 c_{l\beta} + m_T \rho b^2 c_{m\beta})U^2/d$, and $u = \beta$ is the flap deflection. These parameters are defined in Ref. 2. In Eq. (1), subsystems A and B present the dynamics of plunge displacement and pitch angle, respectively. The sliding surface of subsystem A (containing the subtarget of plunge displacement) is defined as $s_1 = \dot{x}_1 + \lambda_1 x_1 = x_2 + \lambda_1 x_1$. The sliding surface of subsystem B (containing the main control objective of pitch angle) is defined as $s_2 = \dot{x}_3 + \lambda_2 x_3 = x_4 + \lambda_2 x_3$. In the preceding equations, λ_1 and λ_2 are positive constants. An intermediate variable z, representing the information from subsystem A, that is, s_1, is incorporated into s_2. Sliding surface s_2 is modified as[7]

$$s_2 = \lambda_2(x_3 - z) + x_4 \tag{2}$$

This modification reflects that the main target is changed from $x_3 = 0, x_4 = 0$ to $x_3 = z, x_4 = 0$. Here $\dot{s}_2$ can be calculated as $\dot{s}_2 = \lambda_2(\dot{x}_3 - \dot{z}) + \dot{x}_4 = \lambda_2 x_4 - \lambda_2 \dot{z} + f_2 + g_2 u$. In the decoupled sliding surface of Eq. (2), the boundedness of x_3 is guaranteed by letting $|z| \le \bar{z}$, $0 < \bar{z} < 1$, where $\bar{z}$ is an upper bound of $|z|$. For the decoupling control, z is defined as $z = \bar{z} \cdot \mathrm{sat}(s_1/\Phi_z)$, where Φ_z is the boundary layer of s_1 and sat(·) is the saturation function. Because $\bar{z}$ is less than one, z presents as a reduced signal of s_1. When s_1 still has not converged to zero, this signal is reduced to z and then is incorporated into s_2 so that the value of x_3 will be limited.[7] As s_1 decreases, z also decreases. When $s_1 \to 0$, $z \to 0$, and $x_3 \to 0$, then $s_2 \to 0$. The control objective is achieved.

By the SMC, an equivalent controller can be obtained from $\dot{s}_2 = 0$ (Ref. 4), that is, $u_{\mathrm{eq}} = (-\lambda_2 x_4 + \lambda_2 \dot{z} - f_2)/g_2$. Because $\dot{z}$ is an explicit function of u and the precise models of f_2 and g_2 are generally unobtainable, the equivalent controller is always unrealizable. Thus,

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the following ADFSMC system is proposed to approach this equivalent controller.

III. ADFSMC

Consider the following fuzzy control system for the control input u_f:

$$\text{Rule } i\text{: if } s_2 \text{ is } F^i\text{, then } u_f \text{ is } \theta_i \tag{3}$$

where $F^i, i = 1, 2, \ldots, n$, are the labels of the fuzzy sets characterized by the fuzzy membership functions $\mu_{F^i}(\cdot)$ and $\theta_i, i = 1, 2, \ldots, n$, are the adjustable fuzzy singletons. The defuzzification of the output is accomplished by the method of center of gravity:

$$u_f = \frac{\sum_{i=1}^{n} \theta_i \times \xi_i}{\sum_{i=1}^{n} \xi_i} = \boldsymbol{\Theta}^T \xi \tag{4}$$

where $\xi_i = \mu_{F^i}(s_2)$ is the firing weight of the ith rule of Eq. (3),

$$\xi = \left[\xi_1 \Big/ \sum_{i=1}^{n} \xi_i, \xi_2 \Big/ \sum_{i=1}^{n} \xi_i, \ldots, \xi_n \Big/ \sum_{i=1}^{n} \xi_i\right]^T$$

and $\boldsymbol{\Theta} = [\theta_1 \ \theta_2 \ \cdots \ \theta_n]^T$. By the universal approximation theorem, there exists an optimal fuzzy system U_f^* such that $u_f^*(s_2, \boldsymbol{\Theta}^*) = \boldsymbol{\Theta}^{*T} \xi$, where $\boldsymbol{\Theta}^*$ is the time-invariant optimal parameter vector.[9] The minimum approximation error is defined as

$$d(t) = u_f^* - u_{\text{eq}}, \qquad 0 \le |d(t)| \le D \tag{5}$$

where the uncertainty bound D is a positive constant. Unfortunately, this uncertainty bound cannot be measured in practical applications. Thus, a bound estimation $\hat{D}(t)$ will be developed to estimate D. Define the estimation error $\hat{D}(t) = D - \hat{D}(t)$, and then assume the control law takes the form

$$u(s_2, \hat{\theta}, \hat{D}) = u_f(s_2, \hat{\theta}) + u_c(s_2, \hat{D}) \tag{6}$$

where the fuzzy controller u_f is used to mimic the equivalent controller and the compensation controller u_c is used to compensate for the difference between the equivalent controller and the fuzzy controller,

$$u_c = -\hat{D}\,\text{sgn}(g_2 s_2) \tag{7}$$

When Eqs. (5) and (6) are used, $\dot{s}_2$ can be calculated:

$$\begin{aligned} \dot{s}_2 &= \lambda_2 x_4 - \lambda_2 \dot{z} + f_2 + g_2(u_f + u_c) \\ &= g_2\left(u_f - u_f^* + u_c + d\right) \end{aligned} \tag{8}$$

Then, the following theorem can be obtained.

Theorem: Consider the aeroelastic system presented in Eq. (1). The control law is designed as in Eq. (6), in which the fuzzy controller u_f is given in Eq. (4) with the adaptive law given in Eq. (9), and the compensation controller u_c is given in Eq. (7) with the bound estimation law presented in Eq. (10):

$$\dot{\hat{\boldsymbol{\Theta}}} = -\dot{\tilde{\boldsymbol{\Theta}}} = \eta_1 g_2 s_2 \xi \tag{9}$$

$$\dot{\hat{D}} = -\dot{\tilde{D}} = \eta_2 |g_2 s_2| \tag{10}$$

where η_1 and η_2 are positive constants. Then the stability of the system can be guaranteed.

Proof: Define a Lyapunov function as

$$V(s_2, \tilde{\boldsymbol{\Theta}}, \tilde{D}) = (1/2)s_2^2 + (1/2\eta_1)\tilde{\boldsymbol{\Theta}}^T \tilde{\boldsymbol{\Theta}} + (1/2\eta_2)\tilde{D}^2 \tag{11}$$

where $\tilde{\boldsymbol{\Theta}} = \boldsymbol{\Theta}^* - \hat{\boldsymbol{\Theta}}$. Differentiating Eq. (11) with respect to time and using Eqs. (4) and (8) yields

$$\begin{aligned} \dot{V}(s_2, \tilde{\boldsymbol{\Theta}}, \tilde{D}) &= s_2 \dot{s}_2 + (1/\eta_1)\tilde{\boldsymbol{\Theta}}^T \dot{\tilde{\boldsymbol{\Theta}}} + (1/\eta_2)\tilde{D}\dot{\tilde{D}} \\ &= g_2 s_2\left(u_f - u_f^* + u_c + d\right) + (1/\eta_1)\tilde{\boldsymbol{\Theta}}^T \dot{\tilde{\boldsymbol{\Theta}}} - (D - \hat{D})|g_2 s_2| \\ &= g_2 s_2\left(\tilde{\boldsymbol{\Theta}}^T \xi - \hat{D}\,\text{sgn}(g_2 s_2) + d\right) + (1/\eta_1)\tilde{\boldsymbol{\Theta}}^T \dot{\tilde{\boldsymbol{\Theta}}} \\ &\quad - D|g_2 s_2| + \hat{D}|g_2 s_2| \\ &= (1/\eta_1)\tilde{\boldsymbol{\Theta}}^T\left(\eta_1 g_2 s_2 \xi + \dot{\tilde{\boldsymbol{\Theta}}}\right) - \hat{D}|g_2 s_2| + d g_2 s_2 \\ &\quad - D|g_2 s_2| + \hat{D}|g_2 s_2| \\ &= d g_2 s_2 - D|g_2 s_2| \\ &\le 0 \end{aligned} \tag{12}$$

The negative semidefiniteness of the Lyapunov function guarantees that s_2, $\hat{\boldsymbol{\Theta}}$, and $\hat{D}$ are bounded. Letting $L(t) = -d g_2 s_2 \le -\dot{V}(s_2, \tilde{\boldsymbol{\Theta}}, \tilde{D})$ and integrating $L(t)$ with respect to time gives

$$\int_0^t L(\tau)\,\mathrm{d}\tau \le V[s_2(0), \tilde{\boldsymbol{\Theta}}(0), \tilde{D}(0)] - V[s_2(t), \tilde{\boldsymbol{\Theta}}(t), \tilde{D}(t)] \tag{13}$$

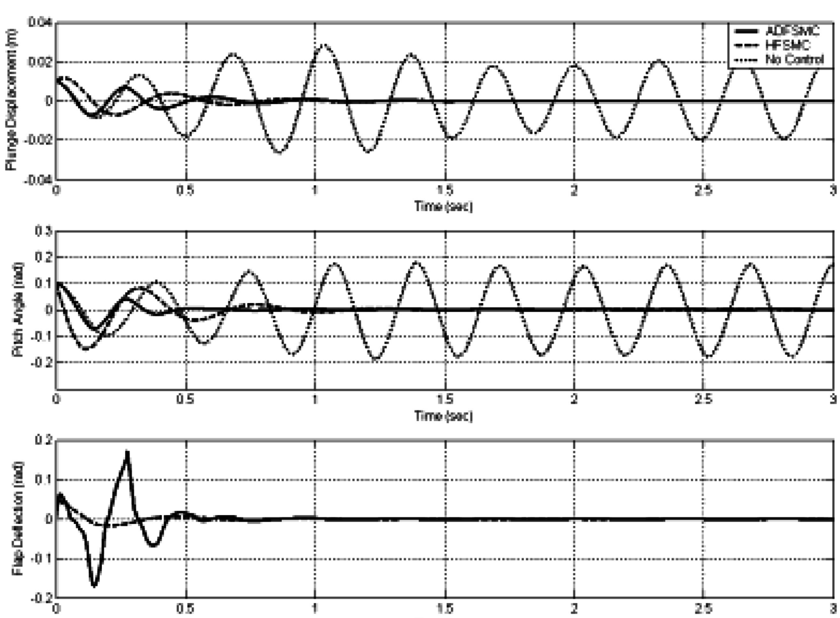

Fig. 1 Responses of an aeroelastic system.

Because $V[s_2(0), \tilde{\Theta}(0), \tilde{D}(0)]$ is bounded and $V[s_2(t), \tilde{\Theta}(t), \tilde{D}(t)]$ is nonincreasing and bounded, it is shown that

$$\lim_{t\to\infty}\int_0^t L(\tau)\,\mathrm{d}\tau < \infty$$

In addition, because $\dot{L}(t)$ is bounded, by Barbalat's lemma (see Ref. 4), it can be shown that $\lim_{t\to\infty} L(t) = 0$. That is, $s_2(t) \to 0$ as $t \to \infty$. Moreover, as discussed in Sec. II, $s_1(t) \to 0$ as $t \to \infty$ can be also achieved. As a result, the control system is asymptotically stable.

IV. Simulation Results

With the parameter values given in Ref. 2 (including the freesbreak tream velocity of $U = 16$ m/s) and for the initial conditions $h(0) = 0.01$ m, $\dot{h}(0) = 0$ m/s, $\alpha(0) = 0.1$ rad, and $\dot{\alpha}(0) = 0$ rad/s, the open-loop responses are shown in Fig. 1. These responses exhibit the limit-cycle oscillations. In Eq. (3), seven triangular-type membership functions are constructed for the antecedent part; the value of the center of the triangular-type membership functions are given as $[-3.5\ \ -1.5\ \ -0.5\ \ 0\ \ 0.5\ \ 1.5\ \ 3.5]$. Then $\bar{z} = 0.2$ and $\Phi_z = 0.25$ are chosen. The simulation results by using the proposed ADFSMC and the hybrid FSMC (HFSMC) presented in Ref. 8 are also shown in Fig. 1. From Fig. 1, it is seen that the proposed ADFSMC can achieve better transient performance than the HFSMC (especially in the main control object of pitch angle response) at the price of larger control effort. This is reasonable for achieving better control performance by using larger control effort. Moreover, the bound estimation $\hat{D}$ is very small, so that the chattering induced by the compensation controller is approximately invisible. The simulation results also show that the proposed ADFSMC achieves better control performance than that presented in Refs. 1–3.

V. Conclusions

An ADFSMC algorithm for a nonlinear aeroelastic system is presented. This control system shows that single flap control can simultaneously drive the plunge displacement and pitch angle to converge to zero. By the derivation of the parameter adaptive laws based on the Lyapunov function, a novel stability analysis of aeroelastic control system is presented. Simulation results have shown that the proposed control algorithm can achieve better control performance than the existing control algorithms for the aeroelastic systems.

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